# Standing waves: which waves fit

Name: Class:

Pluck a guitar string and it plays the same note however hard you pluck it, because the note is set by the length: only certain waves fit. A wave sent along a string reflects at the far end and comes back over itself, and the two, travelling in opposite directions, add up to a pattern that stays in place — a standing wave. By the end you will know which waves fit a string or a pipe, and why, what their frequencies are, and how to measure the speed of sound with one.

Throughout: $L$ is the length of the string or pipe, $\lambda$ the wavelength, $f$ the frequency and $v$ the speed of the waves, so $v = f\lambda$. $f_1$ is the lowest frequency that fits, the first harmonic. A **node** (N) does not move at all; an **antinode** (A) moves the most.

Readings marked **[sim]** come from the simulation; press *Worksheet values* at the start of each part.

## A Two waves that make one standing still

Figure 1 shows two waves at one instant on a string fixed at both ends. They have the same amplitude and the same wavelength; the solid one is travelling to the right and the dashed one to the left.

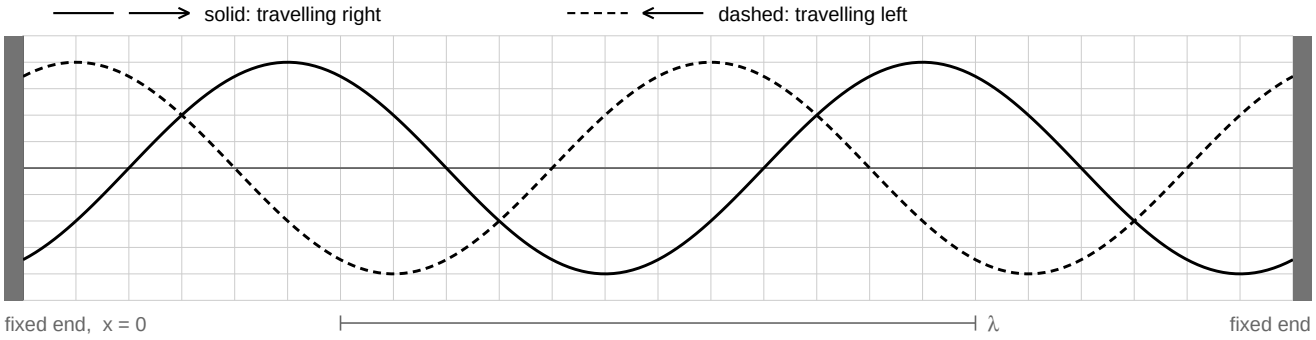



Figure 1 — the two waves at one instant. One small square is 3.5 mm by λ/12.

1. Add them. On each vertical grid line add the two displacements, mark the sum, and join your marks to draw the resultant.
2. Label with N every point where your resultant is zero, including the ends. In terms of λ, neighbouring N are _____ apart.

## B What the ends do

The two waves in Figure 1 are really one wave and its reflection. When a wave reaches a fixed end it comes back upside down; when it reaches an open end of a pipe it comes back the right way up.

1. At the left-hand end of Figure 1 the dashed wave arrives and comes back as the solid one. Read both at $x = 0$: their sum is _____ . Explain why the sum there is the same at every instant, not just this one, so that a fixed end is always a node. **[sim]** *Harmonics*, *String*, *The two travelling waves*: watch the ends.

2. At an open end the wave comes back the right way up. At the end, the incoming and reflected waves then add to _____ , so an open end is always _____ .
3. The air at a closed end of a pipe cannot move, so a closed end is a displacement _____ . But the air arriving there is squashed against the end and then pulled away from it, so the pressure there changes _____ . At an open end the air meets the atmosphere, which holds its pressure steady, so an open end is a pressure _____ .
4. **[sim]** *Harmonics*, *Pipe*, *One closed*, 3rd harmonic, with *Pressure* on. Watch the air on either side of the displacement node inside the pipe, and explain why a displacement node is always a pressure antinode.

## C Which waves fit

Now fit waves between the ends. Figure 2 shows the 1st harmonic of three arrangements of the same length $L$; for the pipes, the curves graph how far the air moves along the pipe. Draw the next two harmonics each can have and fill in the blanks, using $v = f\lambda$ to write $f_1$ in terms of $v$ and $L$. In every pipe drawing, mark with a cross each place where the pressure changes most.

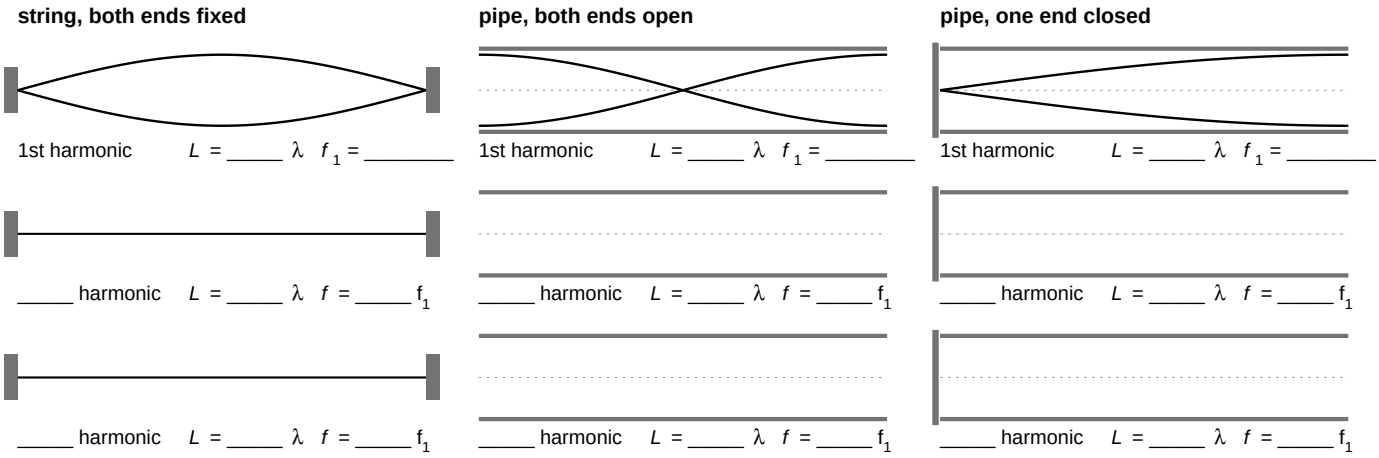



Figure 2 — the ends decide it: a fixed end or a closed end is a node, and an open end is an antinode.

1. **[sim]** *String*: $f_1$ = _____ Hz, $f_2$ = _____ Hz, $f_3$ = _____ Hz. Do they follow your pattern?
2. **[sim]** In the *Sound* panel press *Open pipe, then closed*: the same length of pipe, played twice. Which note is lower, and by how much? Use Figure 2 to explain, and to explain why the closed pipe sounds hollower.

## D Amplitude and phase along the wave

Figure 3 is a string 0.60 m long vibrating in its 3rd harmonic, drawn at a quarter of full size along its length. The dashed lines are the envelope, the furthest each point ever swings; the solid line is the string at one instant.

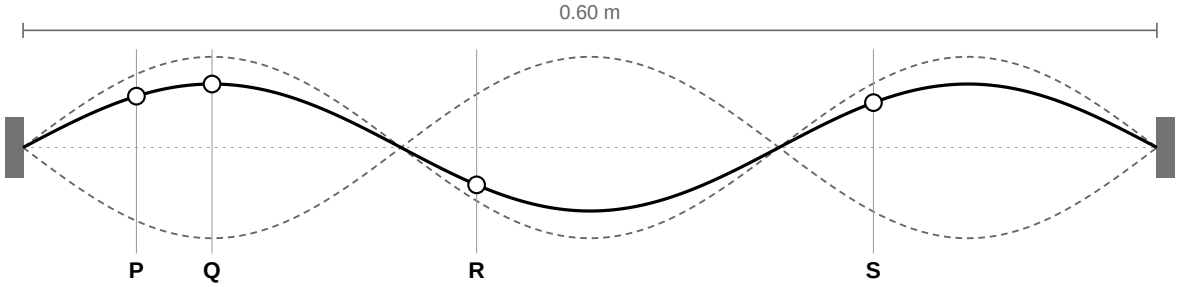



Figure 3 — a quarter of full size along the string. The heights are not to scale, but they are all to the same scale.

1. Measure between two neighbouring nodes: _____ mm on the figure, so _____ m on the string, and λ = _____ m.
2. Measure the height of the envelope above the centre line at each point and fill in the table. Then say whether each point moves in phase with P or in antiphase with it — use the solid line.

| point | $x$ / m | height of envelope / mm | amplitude as a fraction of the largest | in phase or antiphase with P? |
|---|---|---|---|---|
| P | 0.060 | | | — |
| Q | 0.100 | | | |
| R | 0.240 | | | |
| S | 0.450 | | | |

3. **[sim]** Check your table: 3rd harmonic, P at 0.060 m, and Q dragged to each other point in turn.
4. Complete the rule. All the points between two neighbouring nodes move _____ ; two points on either side of one node move _____ .

## E Measuring the speed of sound

A resonance tube stands in water, which closes its lower end; a tuning fork is held over the top and the tube raised until the sound swells. **[sim]** *Pipe*, *One closed*, *Driven*: type 320 into the frequency, choose *amplitude against length L* and press *Sweep L*. Then put the length slider on each peak — its arrow keys move a millimetre at a time.

1. Find the first two resonant lengths $L_1$ and $L_2$ at each frequency and complete the table.

| $f$ / Hz | $L_1$ / m | $L_2$ / m | $L_2 - L_1$ / m | $\lambda$ / m | $v$ / m s$^{-1}$ |
|---|---|---|---|---|---|
| 320 | | | | | |
| 384 | | | | | |
| 512 | | | | | |

2. In the boxes, sketch the air column at the first two resonances, marking N and A. Use your sketches to explain why $L_2 - L_1 = \lambda/2$.

first resonance

second resonance

3. Average your three values of $v$. The simulation's air is at 20 °C: compare your answer with the wave speed it reports.

   mean $v$ = _____ m s$^{-1}$ simulation $v$ = _____ m s$^{-1}$

4. In a real tube the antinode sits a small distance $e$ beyond the open end: the end correction. Explain why $L_2 - L_1$ still gives $\lambda/2$ exactly, when $4L_1$ would not give $\lambda$.

## F Resonance and damping

1. **[sim]** *String*, *Driven*, *amplitude against frequency f*, then *Sweep f*. Sketch the curve on Figure 4, label each peak with its frequency, and explain why the amplitude is large only there.

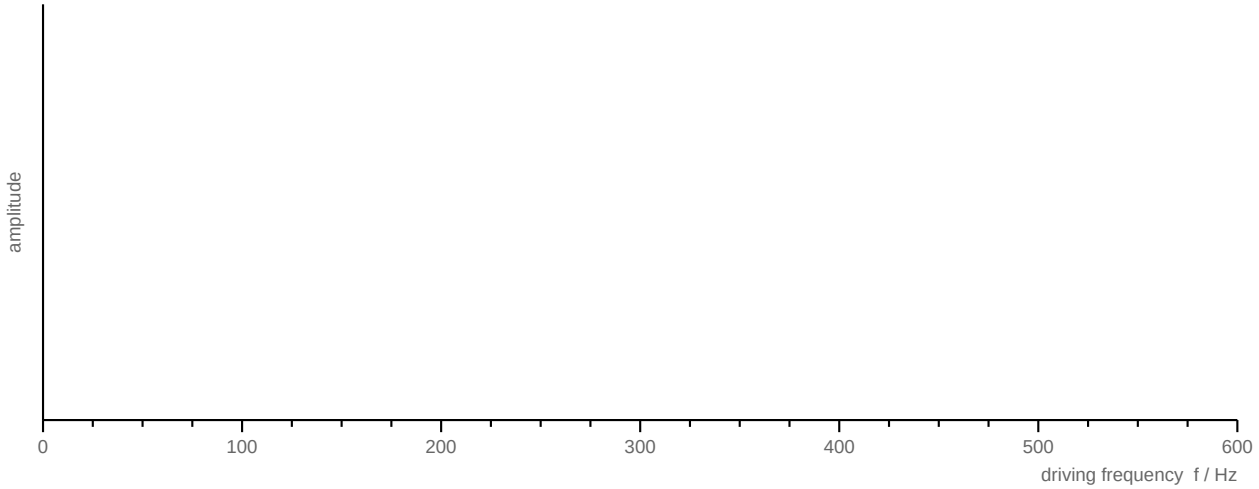



Figure 4 — amplitude of the largest antinode against the driving frequency.

2. Drag the damping slider until a free vibration *halves in* about 10 ms, and sweep again. Sketch the new curve on the same axes, and describe three ways it differs from the first.

3. Type 350.2 into the driving frequency. Press *Light*, then *Set it ringing*, and watch the *Points P and Q* panel. Repeat with *Critical* and with *Heavy*. Sketch displacement against time for each on Figure 5, and under each say how it returns to rest.

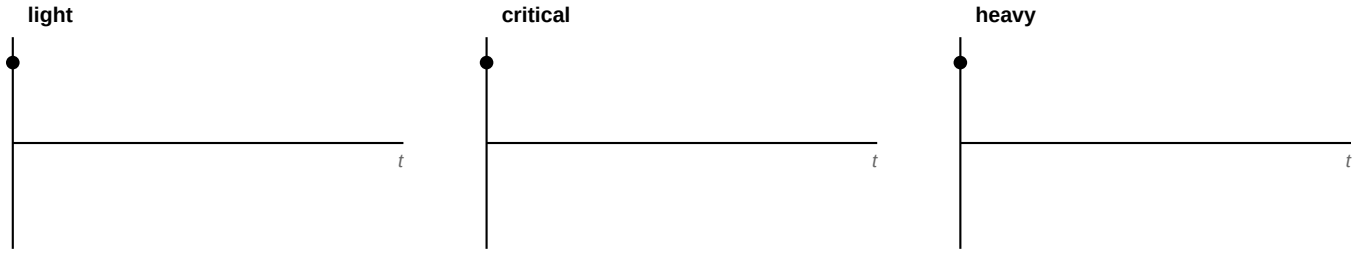



Figure 5 — displacement against time after the driver is taken away.