Answer key — Standing waves

A Two waves that make one standing still

The resultant is a sine curve with the same wavelength as the two waves and, at this particular instant, the same amplitude as each of them: the waves are a sixth of a cycle either side of being in step. It is zero at both fixed ends and at every λ/2 between, so neighbouring nodes are **λ/2** apart.

B What the ends do

At $x = 0$ the dashed wave is 0.87 of its amplitude up, about three and a half squares, and the solid one the same distance down: the sum is **zero**. Reflection at a fixed end turns the wave upside down, so there the reflected wave is always the exact opposite of the one arriving — at every instant, not just this one — and the two always cancel. That is what makes a fixed end a node, and it is the same reason every node stays put: at a node the two travelling waves are always equal and opposite. **Watch for** the answer that the waves "stop". Neither does; each keeps travelling, and it is only their sum that stands still.

Right way up, the two are always equal at the end, so they add to **twice the amplitude of either**: an open end is always an **antinode**.

A closed end is a displacement **node**, where the pressure changes **the most**: a pressure antinode. An open end is a pressure **node**.

Either side of a displacement node the air moves in antiphase, so both sides move towards the node at the same moment, squeezing the air there, and then away from it together, stretching it: the pressure changes most, a pressure antinode. At a displacement antinode the air on both sides moves the same way at once and nothing is squeezed, so it is a pressure node. In the simulation the dots crowd together and spread apart at the displacement nodes, and the pressure curve is largest there.

C Which waves fit

	string L =	f =	open pipe L =	f =	closed pipe L =	f =
1st	½λ	f_1	½λ	f_1	¼λ	f_1
next	λ (2nd)	$2f_1$	λ (2nd)	$2f_1$	¾λ (3rd)	$3f_1$
next	1½λ (3rd)	$3f_1$	1½λ (3rd)	$3f_1$	1¼λ (5th)	$5f_1$

string $f_1 = v/2L$ open pipe $f_1 = v/2L$ closed pipe $f_1 = v/4L$.

The crosses go at every displacement node. Open pipe: at $L/2$ in the 1st harmonic, $L/4$ and $3L/4$ in the 2nd, and $L/6$, $L/2$ and $5L/6$ in the 3rd. Closed pipe: at the closed end in every harmonic, and also at $2L/3$ in the 3rd, and at $2L/5$ and $4L/5$ in the 5th.

116.7, 233.5 and 350.2 Hz: exactly 1 : 2 : 3, because v is the same for every harmonic and λ goes down as $2L$, L, $2L/3$.

The open pipe plays 286.0 Hz and the closed pipe 143.0 Hz: the closed one is **an octave lower**, half the frequency, because only a quarter of a wavelength fits in it rather than a half, so its λ is twice as long. It sounds hollower because it has only the odd harmonics — the second, fourth and sixth are missing from its sound entirely, and they are a good part of what makes the open pipe sound bright.

D Amplitude and phase

Nodes are 50.0 mm apart on the figure, which is 0.200 m on the string, so λ = **0.400 m**. Accept 0.39–0.41 m.

point	x / m	height of envelope / mm	amplitude as a fraction of the largest	in phase or antiphase with P?
P	0.060	9.7	0.81	—
Q	0.100	12.0	1.00	in phase
R	0.240	7.1	0.59	antiphase
S	0.450	8.5	0.71	in phase

Heights are as the table; accept ±0.5 mm. Q is at an antinode, which is why it is the largest.

All the points between two neighbouring nodes move **in phase**, with different amplitudes; points on either side of one node move **in antiphase**, 180° or π out. So in a standing wave the only phase differences are 0 and π, which is the sharpest difference from a travelling wave, where the phase changes steadily along it.

E The speed of sound

f / Hz	L_1 / m	L_2 / m	$L_2 - L_1$ / m	λ / m	v / m s^{-1}
320	0.268	0.804	0.536	1.072	343.1
384	0.223	0.670	0.447	0.894	343.1
512	0.168	0.503	0.335	0.670	343.2

At the first resonance there is a node at the water and an antinode at the top, so $L_1 = \lambda/4$. The next length that fits is $L_2 = 3\lambda/4$, one more node and antinode, and the difference is $\lambda/2$ whatever else is going on at the ends.

Every row gives 343 m s^{-1} to three figures: the simulation's air at 20 °C carries sound at 343.2 m s^{-1}. A student reading to the nearest millimetre will get within about 1 m s^{-1} of it; accept 340–346 m s^{-1}.

The end correction adds the same e to both lengths: $L_1 + e = \lambda/4$ and $L_2 + e = 3\lambda/4$, so subtracting removes it. $4L_1$ would give $\lambda - 4e$, too short. The simulation's pipe has ideal ends, $e = 0$, which is why its $4L_1$ happens to work — worth pointing out as the thing a real tube would not do.

F Resonance and damping

Peaks at 116.7, 233.5, 350.2, 466.9 and 583.6 Hz — the natural frequencies of the string, its harmonics — all 9.3 mm high, with the amplitude dropping to about 0.5 mm, the vibration generator's own movement, between them.

Each harmonic is a natural frequency of the string. Driven at one of them, every reflected wave arrives back at the driver in step with the next push, so each push adds to the energy already there and the amplitude builds until the energy lost to damping each cycle equals the energy put in. Away from them the reflections arrive out of step and largely cancel what the driver does.

Heavier damping: the peaks are **lower** (1.7 mm instead of 9.3 mm), **wider**, and at **slightly lower frequencies** — the first moves from 116.7 to 116.2 Hz, too little to see on a sketch but there in the numbers. The frequencies of the peaks are still, very nearly, the natural frequencies.

Light: it keeps oscillating at its natural frequency, the amplitude falling by the same fraction each cycle. **Critical**: it returns to rest in the shortest possible time without overshooting. **Heavy**: it does not overshoot either, but creeps back more slowly than critical. The two non-oscillating sketches should both start at the same height and never cross the axis, with the critical one reaching it first.